UCLA/95/TEP/39
hep-th/9601082

Exact Relation between Einstein and Quadratic Quantum Gravity[1]

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Abstract

We show the exact equality of the path integral of the general renormalizable fourth order gravitational action to the path integral of the Einstein action coupled to a massive spin-0 field and a massive spin-2 ghost-like field with non-polynomial interactions. The metric in the Einstein version is a highly nonlinear function of the metric in the quadratic version. Both massive excitations are unstable. The respective cosmological constant terms in the two versions can be very different. Some implications are briefly discussed.

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Gravitational actions including higher than linear powers of the Riemann tensor are, for a variety of reasons, interesting both as classical and quantum field theories. They, also, arise generically in (four-dimensional) string one-loop effective actions (see eg ref [1]). The relation between the Einstein-Hilbert action and such nonlinear extensions has been addressed by many authors. One of the earliest results is apparently that of ref. [2]. It establishes that the addition of a term quadratic in the curvature scalar is classically equivalent to the minimal coupling of a scalar field to the E-H action plus a scalar field potential. At the classical level, the most complete considerations were given in [3], [4], where it was shown that the fourth order action involving R^2 and $R^2_{\mu\nu}$ terms is equivalent to the E-H action with a different metric and coupled to a symmetric rank-2 tensor 'matter' field. Equivalence here means that the two actions lead to equivalent equations of motion. These results were reproduced, in a somewhat different formalism, in [5], where it was further demonstrated that the content of the tensor field is presisely that of a massive pure spin-2 ghost-like field, and a massive spin-0 field.

In this paper we consider the quantum theory. Our approach is motivated by that of [3]-[4], but, in the quantum context, we will obtain a rather stronger result. We will derive the exact, albeit formal, equality of the functional integral of the most general quadratic action (eq. (2) below) to that of the Einstein theory with a new metric, and coupled to a massive spin-2 ghost-like field and a massive spin-0 field with nonpolynomial interactions. The metric in the Einstein version turns out to be a highly nonlinear function of the metric in the quadratic theory. It is somewhat remarkable that such an exact transformation of the functional integral, where, of course, one integrates over all metrics without regard of the equations of motion, can be given in closed form. This exact equivalence has various physical implications that we briefly discuss below. The quadratic action is renormalizable, whereas the Einstein action with matter is not (by power counting). Both massive fields turn out to be unstable, the massive spin-2 ghost being actually unstable at tree level. The cosmological constant in the Einstein version can be very different from that in the quadratic action. The same method can be used to examine the relation between other gravitational theories, for example, theories involving arbitrary powers of the Ricci tensor. Here, however, we restrict ourselves to the quadratic action, the protoype for this type of transformation.

Our starting point is the path integral for the general fourth order gravitational theory

$$Z = \int [Dg_{\mu\nu}] \exp\left(i \int d^4x \mathcal{L}(g) \right) \quad , \tag{1}$$

where[3]

$$\mathcal{L}(g) = \sqrt{-g}\,[\,\frac{\gamma}{\kappa^2}R - aR_{\mu\nu}R^{\mu\nu} + bR^2 + \frac{\Lambda}{\kappa^4}\,] \quad . \tag{2}$$

The inclusion of appropriate gauge fixing and associated ghost terms does not affect the derivation below and need not be indicated explicitly. We now introduce an auxiliary, non-propagating field $\chi_{\mu\nu}$ (of mass dimension 2), and rewrite (1)- (2) in the form:

$$Z = \int [Dg_{\mu\nu}][D\chi_{\mu\nu}]C^{1/2}\exp\left(i\int d^4x\mathcal{L}(g,\chi)\right) \quad , \tag{3}$$

where

$$\begin{aligned}\mathcal{L}(g,\chi) = \sqrt{-g}\,[\,\frac{\gamma}{\kappa^2}R \quad &- \quad a(R_{\mu\nu}R^{\mu\nu} - \chi_{\mu\nu}g^{\mu\kappa}g^{\nu\lambda}\chi_{\kappa\lambda}) \\ &+ \quad b(R^2 - (g^{\mu\nu}\chi_{\mu\nu})^2) - \frac{\lambda}{\kappa^2}g^{\mu\nu}\chi_{\mu\nu}\,] \quad .\end{aligned} \tag{4}$$

In (4)

$$\lambda^2 = \Lambda(4b - a) \quad , \tag{5}$$

and $C \equiv Det\left(\sqrt{-g}[a(g^{\mu\kappa}g^{\nu\lambda} + g^{\mu\lambda}g^{\nu\kappa})/2 - bg^{\mu\nu}g^{\kappa\lambda}]\right)$ - such purely local determinants, which are actually equal to unity in dimensional regularization, can be absorbed in the definition of the uncoupled measure $[Dg_{\mu\nu}]$. Integration over $\chi_{\mu\nu}$ gives, of course, back (1)-(2).

We now introduce the quantity

$$\mathcal{R}_{\mu\nu}(g,\chi) \equiv R_{\mu\nu} + \chi_{\mu\nu} \quad , \qquad \mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu} \quad , \tag{6}$$

in terms of which one has

$$\begin{aligned}\mathcal{L}(g,\chi) \quad &= \quad \sqrt{-g}\left[\frac{\gamma}{\kappa^2}\mathcal{R} - a\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + b\mathcal{R}^2\right] \\ &\qquad - \sqrt{-g}\left[\frac{1}{\kappa^2}(\gamma+\lambda)g^{\mu\nu} - 2a\mathcal{R}^{\mu\nu} + 2b\mathcal{R}g^{\mu\nu}\right]\chi_{\mu\nu} \\ &\equiv \quad \hat{\mathcal{L}}(g,\mathcal{R}) - \frac{1}{\kappa^2}\mathcal{F}^{\mu\nu}(g,\chi)\chi_{\mu\nu} \quad .\end{aligned} \tag{7}$$

In (7), we have set

$$\frac{1}{\kappa^2}\mathcal{F}^{\mu\nu}(g,\chi) = \sqrt{-g}\left[\frac{1}{\kappa^2}(\gamma+\lambda)g^{\mu\nu} - 2a\mathcal{R}^{\mu\nu} + 2b\mathcal{R}g^{\mu\nu}\right] \,. \tag{8}$$

[3]We use metric signature (+ - - -), $R_{\mu\nu} = \partial_\nu\Gamma^\lambda_{\mu\lambda} - \ldots$, and $\kappa^2 \equiv 16\pi G$, where G is Newton's constant.

The r.h.s. of (8) defines a composite tensor density field $\mathcal{F}^{\mu\nu}$, which we want to use as a new metric field. To this end we insert unity in the integrand in (3) in the form of a δ-function integration over a tensor density field $\hat{h}^{\mu\nu}$:

$$Z = \int [Dg_{\mu\nu}][D\chi_{\mu\nu}][D\hat{h}^{\mu\nu}] \quad C^{1/2} \prod_{x,\mu,\nu} \delta\left[\sqrt{-h}h^{\mu\nu} - \mathcal{F}^{\mu\nu}(g,\chi)\right]$$
$$\exp\left(i\int d^4x \mathcal{L}(g,\chi)\right) \quad . \tag{9}$$

The tensor field $h^{\mu\nu}$ is, uniquely, defined through $\hat{h}^{\mu\nu} = \sqrt{-h}h^{\mu\nu}$, $h \equiv det\, h_{\mu\nu}$, with $h_{\mu\nu}$ the inverse of $h^{\mu\nu}$. (We could, of course, work throughout in terms of $\hat{h}^{\mu\nu}$, but it is more convenient to express the equations below in terms of $h^{\mu\nu}$.) Now, in the integrand in (9), the δ-function allows one to write:

$$\begin{aligned} \mathcal{L} &= \hat{\mathcal{L}}(g,\mathcal{R}) - \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}\chi_{\mu\nu} \\ &= \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}R_{\mu\nu}(h) + \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}[R_{\mu\nu}(g) - R_{\mu\nu}(h)] \\ &\quad + \hat{\mathcal{L}}(g,\mathcal{R}) - \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}\mathcal{R}_{\mu\nu}(g,\chi) \quad . \end{aligned} \tag{10}$$

Furthermore, one can invert (8) to express $\mathcal{R}(g,\chi)$ in terms of $g_{\mu\nu}$ and $\mathcal{F}^{\mu\nu}(g,\chi) = \sqrt{-h}h^{\mu\nu}$. The result is

$$\begin{aligned} \mathcal{R}_{\mu\nu} = -\frac{1}{2}\frac{1}{\kappa^2}(4b-a)^{-1}(\gamma+\lambda)g_{\mu\nu} \quad &- \quad \frac{1}{2a}\frac{1}{\kappa^2}\frac{1}{\sqrt{-g}}\Big[\sqrt{-h}h^{\alpha\beta}g_{\alpha\mu}g_{\beta\nu} \\ &-b(4b-a)^{-1}\sqrt{-h}h^{\alpha\beta}g_{\alpha\beta}g_{\mu\nu}\Big] \quad , \end{aligned} \tag{11}$$

and may be substituted in (10) to express $\mathcal{L}$ entirely in terms of $g_{\mu\nu}$ and $h_{\mu\nu}$. One finds

$$\begin{aligned} -\mathcal{V}(g,h) &\equiv \hat{\mathcal{L}}(g,\mathcal{R}) - \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}\mathcal{R}_{\mu\nu}(g,\chi) \\ &= -\frac{1}{\kappa^4}(\gamma^2-\lambda^2)(4b-a)^{-1}\sqrt{-g} + \frac{1}{\kappa^4}\frac{\gamma}{2}(4b-a)^{-1}\sqrt{-h}h^{\alpha\beta}g_{\alpha\beta} \\ &\quad + \frac{1}{\kappa^4}\frac{1}{4a}\sqrt{-h}\sqrt{\left(\frac{-h}{-g}\right)}\left[h^{\mu\nu}h^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta} - b(4b-a)^{-1}\left(h^{\alpha\beta}g_{\alpha\beta}\right)^2\right] . \end{aligned} \tag{12}$$

Also, using a standard formula of Riemannian geometry for the difference between the Ricci tensors of two different metrics $g_{\mu\nu}$ and $h_{\mu\nu}$, one obtains

$$\mathcal{L}_{kin} \equiv \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}[R_{\mu\nu}(g) - R_{\mu\nu}(h)]$$

$$
\begin{aligned}
&= \frac{1}{4}\frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}g^{\rho\sigma}g^{\kappa\lambda}\left[2\nabla_\kappa g_{\sigma\nu}\nabla_\rho g_{\mu\lambda} - 2\nabla_\kappa g_{\sigma\nu}\nabla_\lambda g_{\mu\rho}\right.\\
&\quad \left. + \nabla_\kappa g_{\rho\sigma}\nabla_\lambda g_{\mu\nu} - 2\nabla_\kappa g_{\rho\sigma}\nabla_\nu g_{\mu\lambda} + \nabla_\mu g_{\rho\lambda}\nabla_\nu g_{\sigma\kappa}\right]\\
&\quad +(\text{total divergence}) \quad , \qquad (13)
\end{aligned}
$$

with covariant derivatives ∇ computed with the metric $h_{\mu\nu}$. From (10), (12) and (13) one sees that the only remainng dependence on $\chi_{\mu\nu}$ in (9) is in the argument of the δ-function, where, from (6), (8), $\chi_{\mu\nu}$ enters linearly. Assuming, as usual, that one may interchange the order of integrations in the functional integral (9), one may now integrate over $\chi_{\mu\nu}$ to obtain

$$
Z = \int [Dg_{\mu\nu}][D\hat{h}^{\mu\nu}]C^{-1/2}\exp\left(i\int d^4x\mathcal{L}(g,h)\right) \quad , \qquad (14)
$$

where

$$
\mathcal{L}(g,h) = \frac{1}{\kappa^2}\sqrt{-h}h^{\mu\nu}R_{\mu\nu}(h) + \mathcal{L}_{kin}(g,h) - \mathcal{V}(g,h) \quad , \qquad (15)
$$

with $\mathcal{L}_{kin}$, $\mathcal{V}$ given by (13), (12). (15) is the Hilbert-Einstein Lagrangian for the metric $h_{\mu\nu}$ coupled to the 'matter' field $g_{\mu\nu}$, and fully reproduces the results of [3],[4], but now obtained as an exact transformation of the functional integral. To extract the spin content of the rank-2 tensor field $g_{\mu\nu}$, we decompose it into its trace and traceless components with respect to $h_{\mu\nu}$:

$$
\phi \equiv \frac{1}{4}\frac{1}{\kappa}h^{\mu\nu}g_{\mu\nu} \quad , \qquad \frac{1}{\kappa}g_{\mu\nu} = \phi_{\mu\nu} + h_{\mu\nu}\phi \ . \qquad (16)
$$

Then we write

$$
g_{\mu\nu} = \kappa\phi I_\mu^{\ \beta}h_{\beta\nu} \,, \quad g^{\mu\nu} = \kappa^{-1}\phi^{-1}h^{\mu\alpha}I^{-1}{}_\alpha^{\ \nu} \,, \quad g = \kappa^4\phi^4 I h \ , \qquad (17)
$$

with

$$
I_\mu^{\ \nu} \equiv \left[\delta_\mu^\nu + \phi^{-1}\phi_{\mu\alpha}h^{\alpha\nu}\right] \,, \quad I^{-1}{}_\mu^{\ \alpha}I_\alpha^{\ \nu} = \delta_\mu^\nu \,, \quad I \equiv det I_\mu^{\ \nu} \ . \qquad (18)
$$

Substituting the decomposition (16),(17) in (12) one obtains (working now in terms of the 'matter' fields $\phi_{\mu\nu}$, ϕ all indices are raised and lowered by the metric $h_{\mu\nu}$):

$$
\begin{aligned}
\mathcal{V} &= -\frac{1}{\kappa^4}\frac{v_0}{\gamma}\Lambda\sqrt{-h} + \frac{\gamma}{\kappa^2}v_0^{-1}(4b-a)^{-1}\sqrt{-h}\left(\phi - \frac{1}{\kappa}v_0\right)^2\\
&\quad + \frac{1}{\kappa^4}(4b-a)^{-1}\sqrt{-h}(I^{-1/2}-1) + \frac{\gamma}{\kappa^2}v_0^{-1}(4b-a)^{-1}\sqrt{-h}(I^{1/2}-1)\phi^2\\
&\quad - \frac{1}{\kappa^4}\frac{1}{4a}\sqrt{-h}I^{-1/2}\phi^{-2}\phi_{\mu\nu}\phi^{\mu\nu} \,, \qquad (19)
\end{aligned}
$$

with

$$v_0 \equiv \left[\gamma(1-\lambda^2/\gamma^2)\right]^{-1} \ . \tag{20}$$

Note that, since

$$I^{1/2} = 1 - \frac{1}{4}\phi^{-2}\phi_{\mu\nu}\phi^{\mu\nu} + \ \cdots \ , \tag{21}$$

the last three terms in (19) contain linear and higher powers of $\phi_{\mu\nu}\phi^{\mu\nu}$. Accordingly, we shift

$$\phi = \frac{1}{\kappa}v_0(1+\kappa\varphi) \ , \qquad \phi_{\mu\nu} = v_0\varphi_{\mu\nu} \ , \tag{22}$$

so as to absorb the linear term in ϕ in $\mathcal{V}$ in (19). $\mathcal{V}$ provides then the nonvanishng background value for ϕ which is necessary for consistency, since $g_{\mu\nu}$ must possess an inverse. We also conveniently scaled the fluctuating fields φ, $\varphi_{\mu\nu}$ so as to give standard normalization to their kinetic terms in $\mathcal{L}_{kin}$. Thus

$$g_{\mu\nu} = v_0 h_{\mu\nu} + \kappa v_0(\varphi h_{\mu\nu} + \varphi_{\mu\nu}) \ , \qquad \text{and} \qquad I_\mu^{\ \nu} = \left[\delta_\mu^{\ \nu} + \kappa\frac{\varphi_{\mu\alpha}h^{\alpha\nu}}{(1+\kappa\varphi)}\right] \ . \tag{23}$$

Inserting (23) in (14), (15), (19), (13), we finally obtain:

$$Z = \int [D\varphi_{\mu\nu}][D\varphi][D\hat{h}^{\kappa\lambda}]C^{-1/2}\exp\left(i\int d^4x\mathcal{L}(\{\varphi\},h)\right) \ , \tag{24}$$

$$\begin{aligned}
\mathcal{L}(\{\varphi\},h) \ = \ & \frac{1}{\kappa^2}\sqrt{-h}R(h) + \frac{1}{\kappa^4}\frac{v_0}{\gamma}\Lambda\sqrt{-h} \\
& + \frac{3}{2}\sqrt{-h}h^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi - \sqrt{-h}\nabla_\mu\varphi\nabla_\nu\varphi^{\mu\nu} - \frac{1}{2}m_0^2\sqrt{-h}\varphi^2 \\
& + \sqrt{-h}\left[-\frac{1}{4}h^{\mu\nu}\nabla_\mu\varphi_{\alpha\beta}\nabla_\nu\varphi^{\alpha\beta} + \frac{1}{2}\nabla_\nu\varphi^{\mu\alpha}\nabla_\mu\varphi_\alpha^{\ \nu}\right] + \frac{1}{4}m^2\sqrt{-h}\varphi_{\mu\nu}\varphi^{\mu\nu} \\
& + \mathcal{L}_{kin}^I - \mathcal{V}^I \ ,
\end{aligned} \tag{25}$$

with

$$m_0^2 = 2\gamma v_0(4b-a)^{-1}\frac{1}{\kappa^2} \ , \qquad m^2 = \left[\frac{1}{a} + \frac{v_0}{\gamma}\Lambda\right]\frac{1}{\kappa^2} \ . \tag{26}$$

In (25) we wrote out explicitly only the parts of $\mathcal{L}_{kin}$ and $\mathcal{V}$ that are bilinear in the fields φ, $\varphi_{\mu\nu}$. $\mathcal{L}_{kin}^I$, and $\mathcal{V}^I$ then denote the non-polynomial interaction terms containing the trilinear and higher couplings in φ, $\varphi_{\mu\nu}$ from the expansion of $\mathcal{L}_{kin}$, and $\mathcal{V}$, resp., in powers of φ, $\varphi_{\mu\nu}$ upon insertion of (23) in (13), (19). (The unexpanded expressions (13), (19), with the replacements (22), (23), give this nonpolynomial Lagrangian in closed form.)

We have then obtained the exact transfomation of the path integral of the general fourth order theory (1)-(2) to the form (24)-(25). Note that any convenient gauge-fixing term, e.g. $(\Box\partial_\mu g^{\mu\nu})^2$, plus associated FP-ghost terms in (1) is tacitly carried along in the above derivation, and is, at the very end, reexpressed through (23) in terms of φ, $\varphi_{\mu\nu}$ and $h_{\mu\nu}$. A convienient choice of gauge in (1)-(2) will, of course, not translate, in general, into a convenient gauge for computations in terms of the metric $h_{\mu\nu}$ in the theory in the form (24)-(25). According to the standard FP argument, however, the path integral Z is actually independent of the gauge-fixing term choice, so, once the equivalence of (1) to (24) is obtained in one gauge, one may change this gauge to any other in (24)-25), i.e. the equivalence holds independently of the gauge choice. The same is, of course, true for any gauge invariant correlation functions.

Now (25) is the Hilbert-Einstein action for the metric $h_{\mu\nu}$ with a cosmological term and coupled to a massive spin-2 field $\varphi_{\mu\nu}$, and a massive spin-0 field φ. Note that the $\varphi_{\mu\nu}$ kinetic plus mass terms in (25) are not in (the curved-space generalization of) the canonical Pauli-Fierz form[4]. But they are an equally good formulation of the standard Fierz description of a pure spin-2 field, i.e. traceless $\varphi_{\mu\nu}$, and $\nabla_\mu\varphi^\mu{}_\nu$ obeying a constraint equation, which follows from the equations of motion, and involves only *first* derivatives of the fields $\varphi_{\mu\nu}, \varphi, h_{\mu\nu}$[5] [6]. The non-polynomial interaction terms ensure full gauge invariance and continued absence of a spin-1 component beyond the (curved-space) linearized approximation. The lagrangian (25) thus provides a realization of a complete, consistent coupling of a massive spin-2 field to a gravitational background, a noteworthy fact ([7], [5]).

The spin-2 and spin-0 field kinetic terms in (25) come with opposite signs. The overall sign is set by the (Newtonian limit of the) EH term, which makes the spin-2 have the wrong sign, ie be a ghost-like field. We thus find, in the full non-linear theory, the same field content as in the linearized approximation to (2) [8]. Note, however, that in the full theory this can only be achieved by introducing a new metric field which is a highly nonlinear function of the metric in (2); the exact relation between (2) and (25) is non-perturbative.

It is natural to view (2) as the theory formulated in terms of field variables suitable

[4]To go over to that form, express (25) in terms of the traceful field $\psi \equiv \varphi h_{\mu\nu} + \varphi_{\mu\nu}$. The bilinear parts of (25) are then precisely the massive spin-2 action of ref. [8], which, as shown there, can be brought to the Pauli-Fierz mass form by a somewhat different field decomposition of $\psi_{\mu\nu}$. The resulting scalar field is a mixture of our scalar field and $\nabla_\mu\nabla_\nu\varphi^{\mu\nu}$. Note that this affects the definition of the scalar mass term. We do not discuss such alternative formulations since they are not pertinent to our main point here.

[5]The tracelessness of $\varphi_{\mu\nu}$ must be remembered when performing the variation.

for the UV regime (energies higher than Planck scale energy), and (25) as the theory in terms of variables appropriate to the IR region (energies at or below Planck scale). Indeed, recall that (2) is a renormalizable [8], and in fact asympotically free (in the coupling $\alpha^2 \equiv 1/a$) lagrangian [9], [10], [11]. Its loop perturbative expansion is, therefore, applicable in the deep UV region. (25), on the other hand, has, by power counting, the usual non-renormalizable behavior of the Einstein theory coupled to matter.

It is not clear, in the absence of explicit computations, how the renormalizability of (2) appears in (25). The 'matter' fields circulating in loops must serve as regulators. Still, since the metric in (25) is a highly nonlinear composite field in terms of that in (2), simple direct order-by-order cancellation of the nonrenormalizable divergences in the loop expansion of (25) presumably does not occur. Rather, one expects that the divergences are cancelled on mass-shell upon nonlinear shifts of field variables, and/or resummation of appropriate subclasses of graphs. In any case, we seem to have an interesting example of the equivalence of a renormalizable to a (by power counting) non-renormalizable lagrangian.

The Einstein version (25) separates out the massless graviton, which dominates at large distance scales, from the massive fields, and hence is suitable for consideration of the IR regime. It is here that the difficult dynamical issues of the S-matrix asymptotic states and unitarity become relevant. At tree level, the massive fields have masses naturally of the order of the Planck mass. Now, the asymptotically free coupling α grows large in the IR, and indeed tends to diverge below the Planck scale. Since the mass of the spin-2 particle grows with it, this leads to the possibility that this particle disappears from the spectrum at large distances; ie there is "confinement" of the massive spin-2 ghost-like excitation, as has often been suggested in the literature. In any event, at the very least, the following situation should apply. With α large, the Compton wavelength of the massive spin-2 becomes comparable or less than its Schwarzschild radius. This suggests that, by well-known results [12], already at the *classical* level, collapse of the surrounding spacetime and formation of a trapped surface must occur. In such a case, the usual expansion about the tree level description of a bare particle propagating on some given background is clearly not meaningful. Rather, the 0-th order approximation must already include enough interaction effects to correctly describe the appearance of such a highly dressed object, essentially a mini black hole. An even more basic question is that of the stability of these massive particles, ie whether they can appear in the true asymtotic states at all. Inspection of the interaction terms in the Lagrangian (25) shows that *both the spin-2 and the spin-0 particles are unstable*: there are trilinear vertices in $\mathcal{L}_{kin}$ that, for $b \geq 9a/4$, allow the tree-level decay of the $\varphi_{\mu\nu}$-particle into two φ-particles (plus gravitons); and both particles can decay into

gravitons through radiative loop corrections. Now the S-matrix can, strictly speaking, be defined only between in- and out-states containing solely stable particles. In the standard field theoretic treatment of unstable particles [13], this S-matrix connecting stable particles only is constructed in terms of complete, dressed propagators for the unstable particles; and can then be shown [13] to be unitary and causal. This formalism must be applied here too. In fact, the spin-2 particle appears as the simplest example of an unstable particle, ie decaying at tree level (so it *has* to be treated in terms of dressed propagators), except for the fact that its bare propagator has negative residue. In this connection it might be also useful to recall that, in the quantum theory, a negative residue can be traded for negative energy flowing through the propagator. But in a gravitational field there is no invariant meaning of local energy density. So there is nothing immediately inconsistent in allowing localized negative energies, corresponding to the occurance of an unstable excitation, if they do not affect the asymptotic values of the fields.

Another noteworthy feature of the equivalence of (2) to (25) is the relation between their respective cosmological terms. Both vanish if Λ is fine-tuned to zero. For $\Lambda \neq 0$, however, the cosmological constant $v_0\Lambda/\gamma$ in (25) can be very different from Λ. If, in particular, the couplings run appropriately in the IR, it can tend to zero at large scales, even for large values of Λ. It is easily seen that there is more than one senario for the renormalization group flows of the couplings γ, a, b, Λ that could lead to this behavior. The asymptotic freedom of $1/a$ is firmly established, but the present state of the computations of the renormalization of the other couplings, [10], [11], does not yet allow one to draw any definitive conclusions. More investigation is needed to ascertain if this interesting possibility is actually realized.

The method presented here can be used to examine the relation between other gravitational theories. One obvious application is to the supersymmetric version of (2) (cp ref [1]). A more intriguing case is that of more general theories involving arbitrary polynomials in the Ricci tensor. Results will be presented elsewhere.

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